Formulas, Identities, and Formulas

Compiled in the Spring of 2017

UNIT 1

Angle equivalents: $360° = 2\pi \text{ radians} = 1 \text{ revolution} = 400 \text{ gradients}$

$1° = 60' \ (minutes)$ $\quad$ $1' = 60" \ (seconds)$

Degrees to Radians: multiply by $\frac{\pi}{180°}$

Radians to Degrees: multiply by $\frac{180°}{\pi}$

Arc Length Formula: in radians: $s = r\theta$ $\quad$ in degrees: $s = \frac{\pi r\theta}{180°}$

Statute (Land) Mile: 5280 feet, or any equivalent measure

Nautical Mile: the length of an arc on an angle measuring one minute $(1')$

UNIT 2

Sine and Cosine Functions (and transformations)

Sine/Cosine Function: $f(x) = A\sin[B(\theta \pm C)] \pm D$, where:

A is the amplitude $\left(\frac{max-min}{2}\right)$ $\quad$ B is the frequency $\left(\text{Period} = \frac{2\pi}{B}\right)$

C is the phase shift $\quad$ D is the vertical shift $\left(\frac{max+min}{2}\right)$

$+C$ is a shift to the left $\quad$ $+D$ is a shift up
$-C$ is a shift to the right $\quad$ $-D$ is a shift down

Tangent Function: $f(x) = A\tan[B(\theta \pm C)] \pm D$, where:

same as Sine/Cosine ***except****…* $\quad$ B is the frequency $\left(\text{Period} = \frac{\pi}{B}\right)$

UNIT 3

Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$ $\quad$ $\cos\theta = \frac{1}{\sec\theta}$ $\quad$ $\tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\quad$ $\sec\theta = \frac{1}{\cos\theta}$ $\quad$ $\cot\theta = \frac{1}{\tan\theta}$

Ratio Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$ $\quad$ $\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = \tan^2\theta$ $\quad$ $1 + \tan^2\theta = \sec^2\theta$ $\quad$ $1 + \cot^2\theta = \csc^2\theta$

Complimentary Identities

$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$ $\quad$ $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$ $\quad$ $\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$

$\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$ $\quad$ $\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$ $\quad$ $\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$

Even-Odd Identities

$\sin(-\theta) = -\sin\theta$ $\quad$ $\cos(-\theta) = \cos\theta$ $\quad$ $\tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$ $\quad$ $\sec(-\theta) = \sec\theta$ $\quad$ $\cot(-\theta) = -\cot\theta$

Sum Identities

$\sin(x + y) = \sin x\cos y + \cos x\sin y$

$\cos(x + y) = \cos x\cos y - \sin x\sin y$

$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$

Difference Identities

$\sin(x - y) = \sin x\cos y - \cos x\sin y$

$\cos(x - y) = \cos x\cos y + \sin x\sin y$

$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$

Double-Angle Identities

$\sin(2x) = 2\sin x\cos x$

$\cos(2x) = \begin{matrix}\cos^2 x - \sin^2 x \\ 1 - 2\sin^2 x \\ 2\cos^2 x - 1\end{matrix}$

Half-Angle Identities

$\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}}$ $\quad$ $\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}}$ $\quad$ $\tan\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$

UNIT 4

Vectors

Magnitude: length of a vector = $|\vec{v}|$

Component Form: writing a vector as if its terminal point is the origin = $\langle |\vec{v}| \cos\theta\, , |\vec{v}| \sin\theta \rangle$

Dot Product: if $\vec{U} = \langle u_1, u_2 \rangle$ and $\vec{V} = \langle v_1, v_2 \rangle$, then: $\vec{U} \bullet \vec{V} = u_1 v_1 + u_2 v_2$

Orthogonal: perpendicular vectors. Vectors are perpendicular if $\vec{U} \bullet \vec{V} = 0$

Angle between vectors: $\cos\theta = \frac{\vec{U} \bullet \vec{V}}{|\vec{U}||\vec{V}|}$

Unit vector: a vector in component form with a magnitude of one in the same direction as the original vector $\vec{V}$

$$unit\ vector = \langle \frac{x}{|\vec{V}|}, \frac{y}{|\vec{V}|} \rangle$$

Parametric

Parametric Equations: two separate equations that, when paired, create a rectangular graph with a third variable, a parameter, t.

Eliminate a parameter: solve one equation for t, then substitute that expression into the second equation.

Polar

Graphing on a circular grid with an equation set equal to r, the radius.

Equations: $x = r\cos\theta$ $y = r\sin\theta$ $x^2 + y^2 = r^2$ $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Polar Points: $[r, \theta]$ where r is radius and θ is angle measure in either degrees or radians

Multiplication: $[r_1, \theta_1] \cdot [r_2, \theta_2] = [r_1 \cdot r_2, \theta_1 + \theta_2]$

Polar Curves:	Rose Curves	$r = a\sin(n\theta)$ $r = a\cos(n\theta)$	a = length of petals if n is odd, there are n petals if n is even, there are $2n$ petals
	Limaçon	$r = a + b\sin\theta$ $r = a + b\cos\theta$	"dimple" if $a \geq b$ "loop" if $a < b$
	Circles	$r = a\sin(\theta)$ $r = a\cos(\theta)$	where a is the diameter
	Spirals	$r = \theta$	$r = a\theta$ $r = a\theta + b$

Complex Numbers

4 Forms: Rectangular (a, b) Polar $[r, \theta]$
Complex $a + bi$ Trig $(r\cos\theta\, , r\sin\theta\, i)$

DeMoivre's Theorem: $[r, \theta]^n = [r^n, n\theta]$

Complex n^{th} roots Theorem: n^{th} roots of $z = \left[\sqrt[n]{r}, \frac{\theta}{n} + \frac{2\pi k}{n}\right]$

UNIT 5

Triangles

K = the area of a triangle s = semi-perimeter

$K = \frac{bh}{2}$ $K = \frac{1}{2}bc\sin a$ $K = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

Rotary Motion

Angular Velocity – the rate of change of angular position of a rotating body expressed in angle measurement per time (i.e. the speed an object spins in something like radians per second)

Linear Velocity – the resulting distance per time when an angular velocity is applied to a flat surface or another object.

Navigation

Course – a line of travel measured clockwise from North

Bearing – a line of sight measured clockwise from North

UNIT 7

Power Function

Equation: $f(x) = kx^a$

Degree a with at most a zeroes and at most $a - 1$ extrema.

Polynomial Function

Equation: $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$

Remainder theorem – If a polynomial $P(x)$ is divided by $(x - r)$, the remainder is $P(r)$. If $(x - r)$ is a factor of $P(x)$, then $P(r) = 0$.

Multiplicity of zeroes – odd, curve passes through the x-intercept
even, curve touches the x-axis at the zero

Rational Roots – factors of $\frac{p}{q}$, where p is the constant and q is the lead coefficient

Rational Function

Equation: $f(x) = \frac{1}{kx^a}$

Removable Discontinuity (hole): forms if a variable in the denominator reduces to 1

Essential Discontinuity (vert. asymptote): forms if a variable remains in the denominator

End Behavior Asymptote (n = degree of numerator and m = degree of denominator)

- $n < m$ curve approaches $y = 0$
- $n = m$ divide lead coefficients, curve approaches the quotient
- $n > m$ divide entire polynomial. EBA is resulting quotient without remainder

Intermediate Value Theorem

Suppose f is a continuous, real function on an interval, I, and a and b are on I. Then for every $\mathbb{R}y_0$ between $f(a)$ and $f(b)$, there is at least one $\mathbb{R}x_0$between a and b such that $f(x_0) = y_0$

UNIT 8

Exponential Function

Equation: $f(x) = ab^x$ where a is the initial value
growth – if $b > 1$
decay – if $b < 1$

Logistic Function

Equation: $f(x) = \frac{c}{1+ab^x}$ where c is the limit of the curve

Definition of e: $\lim\left(1 + \frac{1}{x}\right) \approx 2.71828$

Logarithmic Function

Equation: $f(x) = \log_b x$ *note*: this curve is the inverse of the exponential function

Logarithm Relations: $\log_b x = y$ is equivalent to the equation $b^y = x$

Logarithm Rules: $\log_b R + \log_b S = \log_b RS$
$\log_b R = \log_b S = \log_b \left(\frac{R}{S}\right)$
$\log_b(x^n) = n \log_b x$

Finance

Compound Interest: $A = P\left(1 + \frac{r}{k}\right)^{kt}$

Continuous Change: $A = Pe^{rt}$

Present Value Annuity: $PV = R\left(\frac{1-(1+i)^{-n}}{i}\right)$

Future Value Annuity: $FV = R\left(\frac{(1+i)^n-1}{i}\right)$