# Pre Calculus

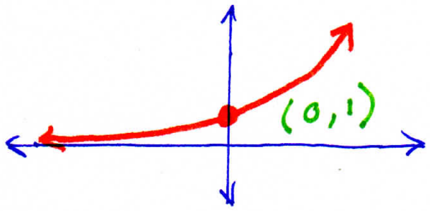
Unit 8 Lesson 1

*Exponential Function Review*

**Recall from the toolkit—exponential functions are of the form $f(x) \overset{=}{\cancel{\neq}} ab^x$**

Compare and contrast the analyses of the following two graphs

$f(x) = 2^x$ (*note* $b > 1$)




Domain $(-\infty, \infty)$

Range $y > 0$

Increasing $(-\infty, \infty)$

Decreasing N|A

Symmetry N|A

Boundedness below

Continuity yes

Extrema N|A

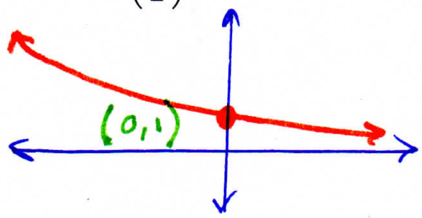
Asymptotes $y = 0$

End Behavior $\lim_{x \to -\infty} f(x) = 0$ | $\lim_{x \to \infty} f(x) = \infty$

Intermediate Behavior

N|A

$f(x) = \left(\frac{1}{2}\right)^x$ (*note* $0 < b < 1$)




Domain $(-\infty, \infty)$

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Boundedness below

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Asymptotes $y = 0$

End Behavior $\lim_{x \to -\infty} f(x) = \infty$ | $\lim_{x \to \infty} f(x) = 0$

Intermediate Behavior

N|A

Compare/Contrast the analyses

The graphs $f(x) = 2^x$ and $f(x) = \left(\frac{1}{2}\right)^x$ are mirror images over the y-axis. Their end behaviors and Inc/Dec domains are inverted (switched).

*Describe how the graph of $f(x) = 2^x$ is transformed to obtain the given function*

1. $g(x) = 2^{x+1}$ trans. 1 unit left

$2^{-x}$

2. $h(x) = 2^{-x}$ reflect over y-axis

* same as $(\frac{1}{2})^x$ !

$= \frac{1}{2^x}$

$= (\frac{1}{2})^x$

3. $k(x) = -3(2^x)$ reflect over x-axis
vert. stretch of 3

Given the function $f(x) = \left(1 + \frac{1}{x}\right)^x$ complete the table and try to find the limit of the function as $x$ approaches infinity

| $x$ | 1 | 10 | 100 | 1000 | 10,000 | 100,000 | 1,000,000 |
|---|---|---|---|---|---|---|---|
| $\left(1+\frac{1}{x}\right)^x$ | 2 | 2.5937 | 2.7048 | 2.7169 | 2.7181 | 2.71827 | 2.71828 |

***Definition***

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e \approx 2.71828$$

**Recall:** When solving exponential equations, the first goal, if possible is to get like bases

Solve: $3^{x-1} = \frac{1}{27}$

$3^{x-1} = 3^{-3}$

$x-1 = -3$

$x = -2$

# Logistic Function

Let a, b, c and k be positive constants with $b < 1$.

* logistic <u>decay</u> if $b > 1$ or $k < 0$

A logistic <u>growth</u> function can written:

$$f(x) = \frac{c}{1+ab^x} \quad \text{OR} \quad f(x) = \frac{c}{1+ae^{-kx}}$$

where constant "c" is the limit of growth

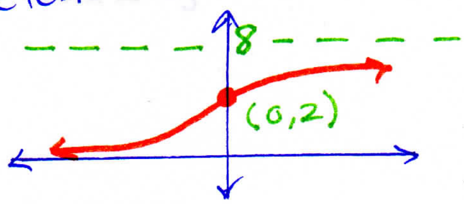
## Ex. 1] Graph $f(x) = \dfrac{8}{1+3(0.7)^x}$

limit of growth = 8

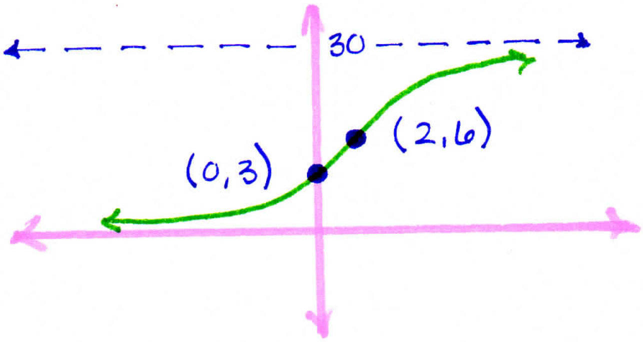
$\lim_{x \to -\infty} f(x) = 0$

y-int = (0,2)

sketch:



Ex. 2] Write the equation of the curve graphed




**Finding "c"**

$f(x) = \dfrac{c}{1+ab^x}$

$f(x) = \dfrac{30}{1+ab^x}$ *

* upper limit

**Finding "a"**

$3 = \dfrac{30}{1+ab^{(0)}}$ **

$3 = \dfrac{30}{1+a}$

$a = 9$

** y-intercept

**Finding "b"**

$6 = \dfrac{30}{1+9b^{(2)}}$ ***

$6 + 54b^2 = 30$

$b^2 = 4/9$

$b = 2/3$

*** other point

$$f(x) = \frac{30}{1+9\left(\frac{2}{3}\right)^x}$$