Pre Calculus

Unit 8 Lesson 2

Exponential and Logistic Modeling

Exponential Growth: $f(x) = ab^x \quad (a > 0,\ b > 1)$

Exponential Decay: $f(x) = ab^x \quad (a > 0,\ 0 < b < 1)$

Ex.1) The population of Bridgetown is growing at a rate of 2.5% per year. The present population is 50,000. If this trend continues, what will the population be in 5 years?

$$f(x) = 50{,}000(1.025)^5$$

$$= 56{,}570 \text{ people}$$

Ex.2) The population of New Zealand was 3,295,000 with an annual average growth rate of 1.4%. Assume this trend continues indefinitely.

a.) Express the population p as a function of n, the number of years after 1985

$$p(n) = 3{,}295{,}000(1.014)^n$$

b.) Predict the population for 1986, 1987, 2004, and 2007

in 1986 = 3,341,130 people

1987 = 3,387,905 people

2004 = 4,291,178 people

2007 = 4,473,942 people

Half-Life: $A(t) = A\left(\frac{1}{2}\right)^{\frac{t}{n}}$ **where,**

A = **initial amount**
$A(t)$ = **final amount**
t = **time**
n = **half life**

Ex.3) Suppose that the half-life of a certain radioactive substance is 20 days. If 5 grams are present initially, how much is left after 57 days?

$A(t) = 5\left(\frac{1}{2}\right)^{57/20}$

$= 0.69$ grams

Ex.4) The half-life of radioactive carbon-14 is 5700 years. What percent of the original amount would you expect to find after 2000 years?

$A(t) = A\underbrace{\left(\frac{1}{2}\right)^{2000/5700}}_{=0.784}$

$\approx 78.4\%$

Exponential Decay:

Ex.5) When kerosene is purified to make jet fuel, pollutants are removed by passing the kerosene through a clay filter. Suppose a filter is fitted in a pipe so that 15% of the impurities are removed for every foot that the kerosene travels.

a.) Write an exponential function to model the percent of impurity left after the kerosene travels *x* feet.

$f(x) = A(0.85)^x$

b.) About what percent of the impurity remains after the kerosene travels 12 feet?

$(0.85)^{12} = 14.2\%$

c.) Will the impurities ever be completely removed? Defend your answer.

NO!
- asym. on x-axis
- cannot mult/divide non-zero numbers and get zero.

Ex.6) Write the exponential model given

a.) Initial value 52, increasing at a rate of 2.3% per day

$f(x) = 52(1.023)^x$

b.) Initial value 5, decreasing at a rate of 0.59% per week

$f(x) = 5(0.9941)^x$

c.) Initial value 250, doubling every 7.5 hours.

$f(x) = 250(2)^{\frac{x}{7.5}}$

Ex.7)

Population of Phoenix (in thousands)

Year	**Population**
1950	107
1960	439
1970	584
1980	790
1990	983
2000	1,321

Use the data in the table and exponential regression to find a model for the population of Phoenix. Predict the population for 2007 using this model. (Use 1900 as $t = 0$)

$f(x) = 20.84(1.045)^{107}$

* "a" and "b" values come from calc. DO NOT round them when typing.

$\approx 2,231,303$ people

Logistic Models:

Recall for logistic functions

$f(x) = \frac{c}{1+ab^x}$ or $f(x) = \frac{c}{1+ae^{-kx}}$

where c is the limit to growth

Ex.8) Suppose that Cary High has approximately 2100 students. Bob, Carol, Ted and Alice start a rumor that spreads logistically according to the formula

$S(t) = \frac{2100}{1+39e^{-0.9t}}$ where $t = 0$ is the day the rumor begins to spread.

a) How many will hear the rumor the day it begins to spread? *(round down to the nearest student)*

$t=0$, $S(t) = \frac{2100}{40}$ $\approx$ 52 students

b) When will 1000 students have heard the rumor?

on Day 4

Ex.9) Write the logistic function that satisfies the given conditions.

Initial population = 16, maximum sustainable population = 128, passing through $(5, 32)$