

LESSON 4

Solving Logs and Exponents

Ex. 1] Solve: $\log_2 x + \log_2 (x+2) = \log_2 (x+6)$

$$\log_2 (x^2 + 2x) = \log_2 (x+6)$$

$$x^2 + 2x = x + 6$$

$$x^2 + x - 6 = 0$$

$$\cancel{x = -3}, x = 2$$

Ex. 2] Solve: $\ln \sqrt{x+2} = 1$

$$\sqrt{x+2} = e^1$$

$$x+2 = e^2$$

$$x = e^2 - 2$$

$$x \approx 5.39$$

Ex. 3] Solve: $20\left(\frac{1}{2}\right)^{\frac{x}{3}} = 5$

$$\left(\frac{1}{2}\right)^{x/3} = \frac{1}{4}$$

$$\left(\frac{1}{2}\right)^{x/3} = \left(\frac{1}{2}\right)^2$$

$$\frac{x}{3} = 2$$

$$x = 6$$

-OR-

$$\left(\frac{1}{2}\right)^{x/3} = \frac{1}{4}$$

$$\left(\frac{1}{4}\right)^{x/6} = \frac{1}{4}$$

$$\frac{x}{6} = 1$$

$$x = 6$$

Ex. 4] Solve $3^x = 17$

$$\log_3 17 = x$$

$$\frac{\log 17}{\log 3} = x$$

$$x \approx 2.58$$

Ex. 5] Solve $e^x - e^{-x} = 10$

$$e^x - \frac{1}{e^x} = 10$$

$$e^{2x} - 1 = 10e^x$$

$$e^{2x} - 10e^x - 1 = 0$$

Let $u = e^x$

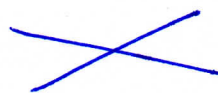
$$u^2 - 10u - 1 = 0$$

$$u = 5 \pm \sqrt{26}$$

$$e^x = 5 \pm \sqrt{26}$$

$$x = \ln(5 + \sqrt{26}) \quad x = \ln(5 - \sqrt{26})$$

$$x \approx 2.31$$



Ex. 6]

$$\frac{2^x + 2^{-x}}{2} = 3$$

$$2^x + 2^{-x} = 6$$

$$2^x + \frac{1}{2^x} = 6$$

$$2^{2x} + 1 = 6 \cdot 2^x$$

$$2^{2x} - 6 \cdot 2^x + 1 = 0$$

Let $u = 2^x$

$$u^2 - 6u + 1 = 0$$

$$u = 3 \pm 2\sqrt{2}$$

$$2^x = 3 + 2\sqrt{2}$$

$$x = \log_2(3 + 2\sqrt{2})$$

$$x \approx 2.54$$

$$x = \log_2(3 - 2\sqrt{2})$$

$$x \approx -2.54$$