

Pre Calculus

Unit 8 Lesson 5

Financial Applications

Compound Interest: Suppose a principal, P , is invested at an annual interest rate, r , compounded k times a year for t years. The amount A in the account after t years is

$$A = P \left(1 + \frac{r}{k}\right)^{kt}$$

Ex.1) John invested \$12,000 at an annual interest rate of 9%. Find the balance after 5 years if the interest is compounded (a) quarterly (b) monthly

$$a) 12,000 \left(1 + \frac{.09}{4}\right)^{20} = \$18,726.11$$

$$b) 12,000 \left(1 + \frac{.09}{12}\right)^{60} = \$18,788.17$$

*

Answers like this should always be close to each other.

Ex.2) Lisa has \$1,500 to invest at a rate of 9.375% interest compounded monthly. How long will it take to double her investment?

$$3,000 = 1500 \left(1 + \frac{.09375}{12}\right)^{12t}$$

$$2 = \left(1 + \frac{.09375}{12}\right)^{12t}$$

$$\log 2 = 12t \log \left(1 + \frac{.09375}{12}\right)$$

$$t \approx 7.42 \text{ months}$$

Continuous Change Model: If interest is compounded continuously,

$$A = Pe^{rt}$$

* Just keep in mind that Lisa would see this on the bank statement for month 8.

Ex.3) Tyler wants to invest \$5,000 at 7.75% compounded continuously so that he can take care of his grandmother when she retires.

(a) How much will he have for his grandmother in 6 years?

$$5,000 e^{.0775(6)} \approx \$7960.07$$

(b) How long will it take for his investment to double?

$$2 = e^{.0775t}$$
$$\ln 2 = 0.0775t$$
$$t \approx 8.94 \text{ years}$$

Comparing Investments: A common basis for comparing investments is the annual percentage yield (APY). This is the percentage rate that, when compounded annually, would yield the same return as the given interest rate with the given compounding period.

Ex.4) A \$2000 investment is made at a bank paying 5.15% annual interest compounded quarterly. What is the equivalent APY? Explain the meaning of your answer.

$$2000(1+r) = 2000\left(1 + \frac{.0515}{4}\right)^4(1)$$

$$1+r \approx 1.0625$$

$$r \approx 0.0625$$

$$r \approx 5.25\% \leftarrow$$

Rate at which you would invest annually to get the same return.

Ex.5) Which is a better investment?

Option A: one that pays 8.75% compounded quarterly or

Option B: one that pays 8.7% compounded monthly

$$1+r = \left(1 + \frac{.0875}{4}\right)^4$$

$$r \approx .090413$$

$$\approx 9.04\%$$

vs.

$$1+r = \left(1 + \frac{.087}{12}\right)^{12}$$

$$r \approx .09055$$

$$\approx \underline{\underline{9.05\%}}$$

better option

Annuity: A series of equal periodic payments.

Future Value of an Annuity: $FV = R \left(\frac{(1+i)^n - 1}{i} \right)$

where R = amount of investment

i = rate per compounding period

n = # of equal periodic payments

Ex.6) At the end of each quarter, Emily makes a \$500 payment into a mutual fund. If her investment earns 7.88% compounded quarterly, what will the value be in 20 years?

$$FV = 500 \left(\frac{(1 + .0197)^{80} - 1}{.0197} \right) \quad * \quad \frac{0.0788}{4} = 0.0197$$
$$= \$95,483.39$$

Present Value: The net amount of money put into an annuity.

$$PV = R \left(\frac{1 - (1+i)^{-n}}{i} \right)$$

Ex.7) Joe purchases a car for \$18,500. What are the monthly payments for a 4-year loan with \$2,000 down if the APR is 2.9%?

$$16,500 = R \left(\frac{1 - (1 + \text{Ans})^{-48}}{\text{Ans}} \right)$$

$$\frac{0.029}{12} = 0.00241\bar{6}$$

* (use Ans option on your calculator)

$$R \approx \$364.49$$